A model of structure and dynamics of solar prominences

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Problem

Solar prominences are magnetic structures incarcerating cool and dense gas in an otherwise hot solar corona. Prominences can be categorized as quiescent and active. Their origin and the presence of cool gas ($\sim 10^4$ K) within the hot ($\sim 10^6 K$) solar corona remains poorly understood. The structure and dynamics of solar prominences was investigated in a large number of observational and theoretical (both analytical and numerical) studies. In this paper, an analytic model of quiescent solar prominence is developed and used to demonstrate that the prominence velocity increases exponentially, which means that some gas falls downward towards the solar surface, and that Alfvén waves are naturally present in the solar prominences. These theoretical predictions are consistent with the current observational data of solar quiescent prominences.

Analytical Approach

We develop an analytical approach to investigate the dynamics of solar quiescent prominences by considering a simple model that is suitable for such an analytical treatment. The main theoretical results obtained from the model are:

- an exponential increase of the prominence velocity within very short time (few minutes) and then resuming the motion with a uniform velocity;
- the downfall of cool gas and neutral material toward the solar surface, which is consistent with the observational data;
- the theoretical evidence for the existence of Alfvén waves responsible for driving oscillations observed in solar quiescent prominences

MHD Equations

Let $\vec{B}(y,t)\hat{z}$ and $\vec{v} = v(y,t)\hat{z}$ be the perturbations in the magnetic field and velocity of the medium respectively. The magnetic field acting on the prominence is described as $\vec{B} = [0, B_0, \vec{B}(y,t)]$, where $\vec{B}(y,t)\hat{z}$ is perturbation in magnetic field, and B_0 is constant magnetic field in the y direction, perpendicular to the sheet.
The z-component of the MHD momentum equation for the medium outside the prominence sheet can be written in the following form,

$$\rho_0 \frac{\partial v}{\partial t} = \frac{B_0}{4\pi}\frac{\partial B}{\partial y}. \tag{1}$$

and the z-component of MHD induction equation becomes,

$$\frac{\partial B}{\partial t} = B_0 \frac{\partial v}{\partial y}. \tag{2}$$

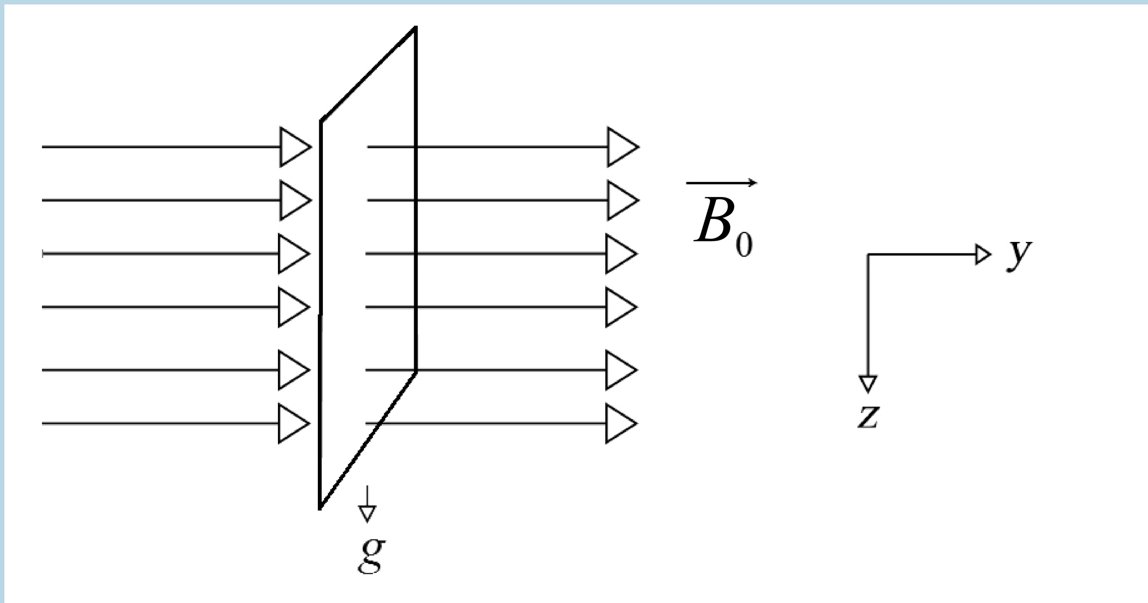
Alfven waves

From Eqs. 1 & 2, the following equations are obtained,

$$\frac{\partial^2 v}{\partial t^2} = \frac{B_0^2}{4\pi\rho_0}\frac{\partial^2 v}{\partial y^2} \tag{3}$$

$$\frac{\partial^2 B}{\partial t^2} = \frac{B_0^2}{4\pi\rho_0}\frac{\partial^2 B}{\partial y^2}. \tag{4}$$

The above two equations (3 & 4) are Alfven Wave equations, with Alfven wave velocity $V_A = \frac{B_0}{\sqrt{4\pi\rho_0}}$.



The following initial conditions are considered. Initial Condition : at $t = 0, B = 0$ and $v = 0$. The boundary Condition : at $y = 0, v = u(t)\Big|_{y=0}$, where $u(t)$ is the prominence sheet's vertical velocity component. The general solution to the above wave equation (3) is

$$B = F(y - V_A t) + G(y + V_A t), \tag{5}$$

The general solution of the one-dimensional wave equation is the sum of a right traveling function F and a left traveling function G. "Traveling function" implies that the shape of these functions (arbitrary) remains invariant with respect to y. However, the functions are translated left and right with time at the speed V_A. The solution, on substitution in eq. 1 & 2, yields the following:

$$v = \frac{1}{\sqrt{4\pi\rho_0}}[-F(y - V_A t) + G(y + V_A t)]. \tag{6}$$

The momentum equation for the prominence sheet reads as:

$$m_p \frac{\partial u}{\partial t}\Big|_{y=0} = m_p g - \int_{y=0-}^{y=0+} \frac{B_0}{4\pi}\frac{\partial B}{\partial y}\Big|_{y=0}. \tag{7}$$

where m_p is the integrated mass density across the thickness of the thin sheet (mass density $\rho \times$ thickness t) and u is the vertical component of prominence sheet's velocity .
The second boundary condition i.e normal component of the magnetic field being continuous across the plate, renders eq. 7 as

$$m_p \frac{\partial u}{\partial t}\Big|_{y=0} = m_p g - \frac{B_0 B}{2\pi}\Big|_{y=0}. \tag{8}$$

Incorporating the initial conditions in 5 & 6, for t=0, we obtain $F(y) = G(y)$. The initial condition $t = 0, v = 0$ yields $F(y) = -G(y)$. Thus,

$$\begin{array}{ll} G(y) = 0 & \\ F(y) = 0 & t = 0, y > 0 \end{array} \tag{9}$$

Now consider, $(y - V_A t) = \xi$ and $(y + V_A t) = \eta$. Similarly, $\xi > 0$ implies $F(\xi) = 0$ and $\eta > 0$ implies $G(\eta) = 0$. Also note, for the 2 cases, $\xi > 0$ implies $y > V_A t$ which in turn, gives us $F(\xi) = 0$ and $G(\xi) = 0$. This is a trivial solution. However, for $\xi < 0$, we have $y < V_A t$ which implies $F(\xi) \neq 0$. This renders equation (8) the following form:

$$\frac{m_p V_A}{\sqrt{4\pi\rho_0}}\frac{dF(-V_A t)}{dt} = m_p g - \frac{B_0 F(-V_A t)}{2\pi}. \tag{10}$$

This is a differential equation of first order, which is solved by using integrating factor (**Saha, 2011**)

Theory explains observations!

Our model (**Saha et.al.**) of solar quiescent prominences was developed and used to determine the velocity of the prominences. The main theoretical prediction of our simple model is that the prominence sheets move at steady uniform downward velocities (few kms/s) within their planes, in agreement with the observations. An important feature of our simple model is the downfall of cool, dense and neutral gas towards the solar surface. The falling gas may generate Alfvén waves, which could potentially drive global and local oscillations observed in solar prominences. The recent observations give strong evidence for the existence of both the oscillations and waves.

References

[1] Saha et.al.: *An analytical model of prominence dynamics*, Advances in Space Research (Elsevier-COSPAR), Vol 60 (8), 2017.

[2] Saha. *Differential Equations: A Structured Approach*, Cognella, 2011.