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An analytical model of prominence dynamics

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Abstract

Solar prominences are magnetic structures incarcerating cool and dense gas in an otherwise hot solar corona. Prominences can be categorized as quiescent and active. Their origin and the presence of cool gas ($\sim 10^4$ K) within the hot ($\sim 10^6$ K) solar corona remains poorly understood. The structure and dynamics of solar prominences was investigated in a large number of observational and theoretical (both analytical and numerical) studies. In this paper, an analytic model of quiescent solar prominence is developed and used to demonstrate that the prominence velocity increases exponentially, which means that some gas falls downward towards the solar surface, and that Alfvén waves are naturally present in the solar prominences. These theoretical predictions are consistent with the current observational data of solar quiescent prominences.

Keywords: Solar physics; Sun; Solar prominence; MHD waves

1. Introduction

It is a well known fact that solar prominences are cool, dense plasma clouds composed of small-scale ever-changing threads of fibrils, embedded in the hot solar corona (Anderson and Athay, 1989; Berger and Ricca, 1996). The prominence plasma is in nearly equilibrium state supported by the magnetic field against gravity (Kippenhahn and Schluter, 1957; Raadu and Kuperus, 1973).

Quiescent prominences are large and appear as thin vertical sheets endowed with fine filamentary structure. These prominences display minor changes over a period of time (days) (Webb et al., 1998). Irrespective of the "quiescent" phrase, these prominences display remarkable mass motion when observed in high resolution Hα movies. These filaments possess the solar material concentrated as rope-like structures with diameter less than 300 km.

Primarily, two types of topology have been suggested for supporting prominences that are related to magnetic fields. The first one was put forward by Kipppenhahn and Schluter in 1957 (Kippenhahn and Schluter, 1957). Kuperus and Tandberg-Hanssen proposed the latter in 1967 (Kuperus and Tandberg-Hanssen, 1967). This was developed further by Kuperus and Raadu (K-R) in 1973 (Raadu and Kuperus, 1973). In the Kuperus-Schlitter (K-S) model, the prominence material sits on top of the field lines supported by the normal polarity field. The K-R model suggests that the prominence is embedded in an inverse polarity field. Simply stated, a prominence is considered as a sheet of plasma, erected in the corona, above a magnetic neutral line.

Prominences are highly dynamical structures exhibiting flows in Hα, UV and EUV lines. The study of these flows improve our understanding of prominence formation and stability, the mass supply and the magnetic field structure of the prominence imparting great interest to these topics.

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A complex dynamics with vertical down flows, up flows and horizontal flows is observed in the Hα lines and quiescent limb prominences (Chae et al., 2008; Engvold et al., 1985; Kubota and and Uesugi, 1986; Lin et al., 2003; Zirker et al., 1998). The velocity of these flows lies between a range of 2 and 35 km/s, while in EUV lines, these flows seem to be of slightly higher velocity. The pertinent aspect of these observations correspond to various temperatures indicating the speed of flow corresponding to different parts of the prominence. These flows seem to be field aligned due to the filament plasma.Vertical filamentary downflows often have been observed in vertically striated or 'hedgerow' prominences (Engvold, 1976; Martres et al., 1981) as well as vortices (Liggett and Zirin, 1984). Explaining these observations of vertical and rotational flows with existing theoretical MHD models is one of the major goals of prominence's investigations.

In more recent numerical studies performed by Terradas et al. (2015), the MHD equations have been solved and time evolution of solar quiescent prominences embedded in sheared magnetic arcades has been investigated. Moreover, Terradas et al. (2016) have studied solar active prominences embedded in magnetic flux ropes. The authors have shown that prominences may originate in the solar photosphere and presented their evolution through the solar atmosphere. The physical properties of the solar prominences and the existence of oscillations associated with such prominences resulting from numerical simulations have also been presented and discussed.

In this paper, we develop an analytical approach to investigate the dynamics of solar quiescent prominences by considering a simple model that is suitable for such an analytical treatment. The main theoretical results obtained from the model are:

- an exponential increase of the prominence velocity within very short time (few minutes) and then resuming the motion with a uniform velocity;
- the downfall of cool gas and neutral material toward the solar surface, which is consistent with the observational data;
- the theoretical evidence for the existence of Alfvén waves responsible for driving oscillations observed in solar quiescent prominences

The paper is organized as follows: Section 2 presents the model of the prominence, based on the MHD equations; this is followed by the obtained results and conclusion in Sections 3 and 4 respectively.

2. MHD equations

Hα photographs of quiescent solar prominences above the solar limb often show evidence of the prominence plasma assuming the form of vertically oriented, narrow filaments (Tandberg-Hanssen, 1995). Vector magnetic fields, using the Hanle effect, have been used to observe the prominence plasmas. Such observations have helped establishing the fact that the magnetic fields inside the prominences are horizontal, binding across slab like macroscopic prominence while the principal field component remains parallel to the horizontal length of the prominence (Leroy, 1989). The narrow vertical filaments are composed of pieces of plasma lined up vertically. Hα observations also demonstrate that the filamentary structures of a quiescent prominence are not truly static Tandberg-Hanssen (1995) and Zirin (1988). Here, we attempt to analyze the dynamics of such a prominence using the MHD equations (Fig. 1).

Suppose there exists a one dimensional infinite vertical rigid sheet of perfectly conducting massive material sitting in a perfectly conducting incompressible static fluid. Let the sheet be threaded by a uniform magnetic field that is perpendicular to the sheet. Gravity is assumed to be uniform and acts vertically to the prominence sheet. Gravity is neglected for the medium (assuming the medium's density to be significantly less compared to the sheet's density) but considered to be acting on the prominence thread. Let $\overrightarrow{B}(y,t)\hat{z}$ and $\overrightarrow{v} = v(y,t)\hat{z}$ be the perturbations in the magnetic field and velocity of the medium respectively. The magnetic field acting on the prominence is described as $\overrightarrow{B} = [0, B_0, \overrightarrow{B}(y,t)]$, where $\overrightarrow{B}(y,t)\hat{z}$ is perturbation in magnetic field, and B_0 is constant magnetic field in the y direction, perpendicular to the sheet.

The z-component of the MHD momentum equation for the medium outside the prominence sheet can be written in the following form,

$$\rho_0 \frac{\partial v}{\partial t} = \frac{B_0}{4\pi} \frac{\partial B}{\partial y}. \quad (1)$$

and the z-component of MHD induction equation becomes,

$$\frac{\partial B}{\partial t} = B_0 \frac{\partial v}{\partial y}. \quad (2)$$

It must be pointed out that we have not used any small amplitude approximation to linearize the MHD equations in order to obtain the above equations.

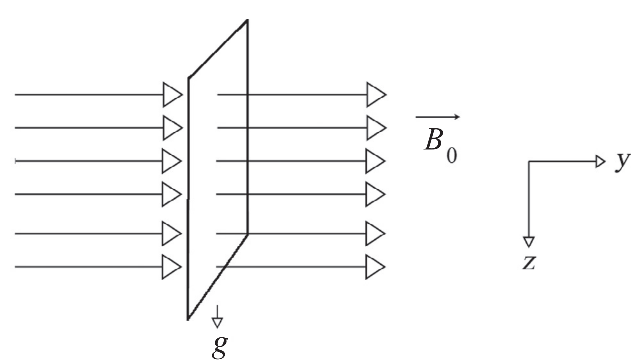

Fig. 1. Schematic representation of prominence sheet along with conditions.

2.1. Alfven wave equations

From Eqs. 1 and 2, the following equations are obtained,

$$\frac{\partial^2 v}{\partial t^2} = \frac{B_0^2}{4\pi\rho_0}\frac{\partial^2 v}{\partial y^2} \tag{3}$$

$$\frac{\partial^2 B}{\partial t^2} = \frac{B_0^2}{4\pi\rho_0}\frac{\partial^2 B}{\partial y^2}. \tag{4}$$

The above two Eqs. (3 and 4) are Alfven Wave equations, with Alfven wave velocity $V_A = \frac{B_0}{\sqrt{4\pi\rho_0}}$.

2.2. Solution to wave equations

The following initial conditions are considered. Initial Condition: at $t = 0, B = 0$ and $v = 0$. The boundary Condition: at $y = 0, v = u(t)|_{y=0}$, where $u(t)$ is the prominence sheet's vertical velocity component.

The general solution to the above wave Eq. (3) is

$$B = F(y - V_A t) + G(y + V_A t), \tag{5}$$

The general solution of the one-dimensional wave equation is the sum of a right traveling function F and a left traveling function G. "Traveling function" implies that the shape of these functions (arbitrary) remains invariant with respect to y. However, the functions are translated left and right with time at the speed V_A (D'Alembert, 1747). The solution, on substitution in Eq. 1 and 2, yields the following:

$$v = \frac{1}{\sqrt{4\pi\rho_0}}[-F(y - V_A t) + G(y + V_A t)]. \tag{6}$$

2.3. Momentum equation for prominence sheet

It is imperative to understand the nature of the velocity in order to lend credence to the theory of the dynamics of the filaments.The momentum equation for the prominence sheet reads as:

$$m_p \frac{\partial u}{\partial t}\bigg|_{y=0} = m_p g - \int_{y=0-}^{y=0+} \frac{B_0}{4\pi}\frac{\partial B}{\partial y}\bigg|_{y=0}. \tag{7}$$

where m_p is the integrated mass density across the thickness of the thin sheet (mass density $\rho\times$ thickness t) and u is the vertical component of prominence sheet's velocity.

The second boundary condition i.e. normal component of the magnetic field being continuous across the plate, renders Eq. (7) as

$$m_p \frac{\partial u}{\partial t}\bigg|_{y=0} = m_p g - \frac{B_0 B}{2\pi}\bigg|_{y=0}. \tag{8}$$

Incorporating the initial conditions in 5 and 6, for t = 0, we obtain $F(y) = G(y)$. The initial condition $t = 0$, $v = 0$yields $F(y) = -G(y)$. Thus,

$$\begin{matrix} G(y) = 0 \\ F(y) = 0 \end{matrix} \quad t = 0, y > 0 \tag{9}$$

Now consider, $(y - V_A t) = \xi$ and $(y + V_A t) = \eta$. Similarly, $\xi > 0$ implies $F(\xi) = 0$ and

$\eta > 0$ implies $G(\eta) = 0$. Also note, for the 2 cases, $\xi > 0$ implies $y > V_A t$ which in turn, gives us $F(\xi) = 0$ and $G(\xi) = 0$. This is a trivial solution. However, for $\xi < 0$, we have $y < V_A t$ which implies $F(\xi) \neq 0$. This renders Eq. (8) the following form:

$$\frac{m_p V_A}{\sqrt{4\pi\rho_0}}\frac{dF(-V_A t)}{dt} = m_p g - \frac{B_0 F(-V_A t)}{2\pi}. \tag{10}$$

This is a differential equation of first order, which is solved by using integrating factor (Saha, 2011) as follows:

$$I = \exp\int (2\rho_0/m_p)dx. \tag{11}$$

where $x = -V_A t$. Applying the integrating factor yields the solution for the Eq. (11) as,

$$u = \frac{m_p g}{2\rho_0 V_A}[1 - exp(-\rho_0 V_A t/m_p)] \tag{12}$$

where,

u is the vertical component of prominence velocity,
m_p is the product of prominence mass density and width of the thread,
g is the acceleration due to gravity of the sun,
ρ_0 is the density of the medium outside the filament,
$V_A$is the Alfven wave velocity
and $t$is the time.

3. Results

The parameters used in our calculations have the following approximated values: the acceleration due to gravity on the solar surface g is 27.42×10^3 cm s^{-2}, the prominence filament density is 10^{11} particles/cm^3 and the coronal density is 10^9 particles/cm^3 (Petrie and Low, 2005), and the filament width is 300 km (Low, 1982). The prominence velocities are determined for different Alfven wave velocities ranging from of 1×10^8 cm s^{-1} to 3×10^8 cm s^{-1}. Plots for different cases are given below (see Fig. 2).

The obtained results show that there is an exponential increase in the velocity of the prominence thread from the equilibrium state over a very short time (few minutes). Then, there is a downward fall (towards solar surface) of the prominence thread with a uniform velocity. Moreover, our results demonstrate the existence of Alfvén waves, which are likely to trigger oscillations commonly observed in solar prominences. The observational data collected by the Solar Optical Telescope on the Hinode satellite (e.g., Tsuneta et al., 2008) are relevant to the theoretical results obtained in this paper because the velocity of prominences

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can be determined from these observations. The data also show the downward fall of a cool and neutral gas. The observed oscillations are either global or local (see Mackay et al., 2010 and references therein), and they are likely to be driven by MHD waves present within the prominences. The prominence velocity vs time is shown in Fig. 2.

Fig. 2 shows plots of the prominence velocity for different Alfven wave velocities. For $V_A = 1000$ km/s, the uniform value of filament velocity is 4.11 km/s, whereas for $V_A = 2000$ km/s it is 2.05 km/s and for $V_A = 3000$ km/s, it is 1.37 km/s. This is in agreement with the observations.

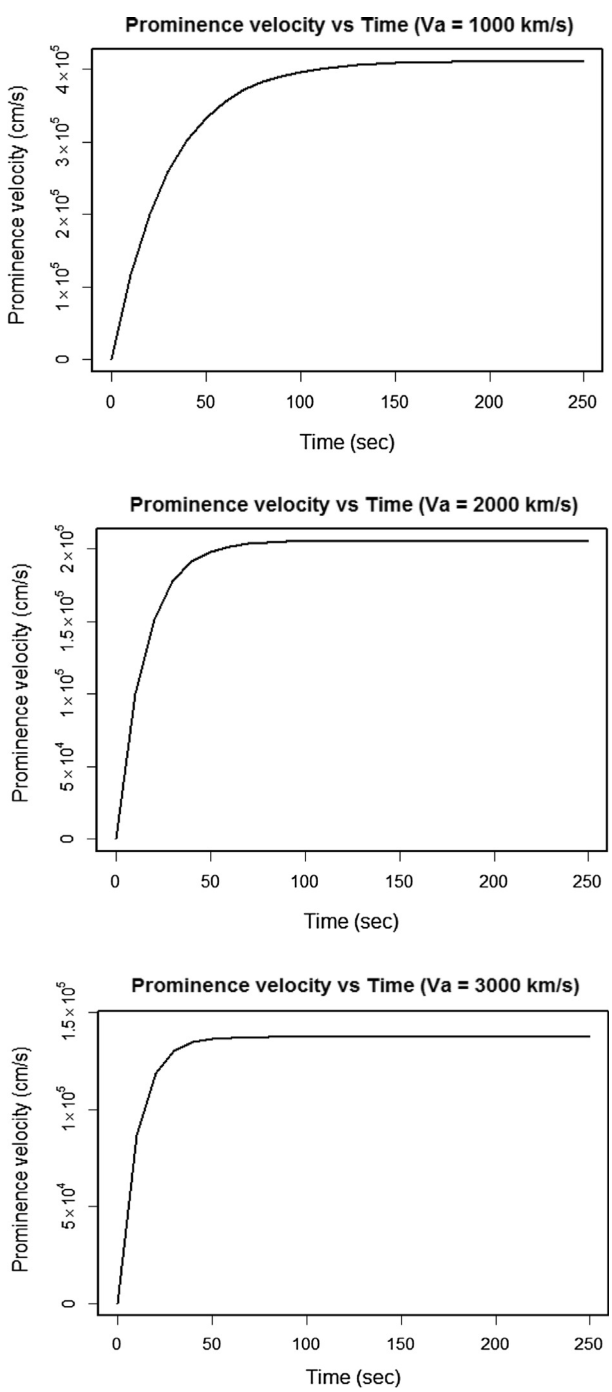

Fig. 2. Prominence velocity as a function of time (Eq. (12)) with Alfven wave velocities of 1000 km s^{-1}, 2000 km s^{-1}, 3000 km s^{-1}.

The vertical down-flow of matter has been reported by Kubota and and Uesugi (1986). They found that the downward motion is predominant in the observed stable filament and ranges up to 5.3 km/s. Engvold (1976) and references therein also reported the overall motion of prominence material is directed downward and measured flow velocities of 5–15 km/s. Berger et al. (2008) found down flows of bright knots less than 10 km s^1 looking at Ca II H images in the line center. Using Hinode/SOT data Schmieder and colleagues (Schmieder et al., 2010) have shown that the horizontal velocity in the quiescent prominences can reach up to 11 km s^{-1}.

Our results also show Alfven waves (not necessarily small amplitude, as we did not apply linearized approximations) could be produced by the prominence filament's vertical motion through the uniform background magnetic field. These waves are mostly localized (as for $y > V_A t$ implies $F(\xi) = 0$ and $G(\xi) = 0$, i.e. no waves can propagate beyond the distance $V_A t$ from the prominence axis). As already mentioned above, the waves drive global or local oscillations, whose presence in solar quiescent prominences has been confirmed observationally, (Mackay et al., 2010) and references therein.

4. Conclusion

A simple MHD model of solar quiescent prominences was developed and used to determine the velocity of the prominences. The results obtained from this model were then compared to the available solar observations. The main theoretical prediction of our simple model is that the prominence sheets move at steady uniform downward velocities (few kms/s) within their planes, in agreement with the observations. An important feature of our simple model is the downfall of cool, dense and neutral gas (it must be neutral, so it can fall through approximately horizontal magnetic field lines) towards the solar surface. The falling gas may generate Alfvén waves, which could potentially drive global and local oscillations observed in solar prominences. The recent observations (see Sections 1 and 3) give strong evidence for the existence of both the oscillations and waves.

Based on the above, we conclude that our simple analytical model of solar quiescent prominences describes at least some aspects of the dynamics of prominence, which is consistent with the available observed data. The model does have some predictive power, which is obviously limited because of the simplicity of our model. Nevertheless, the results of this paper may set up the baseline for future analytical work on solar prominences. In the near future, we intend to consider the localized bow like structure of magnetic field (Low, 1982) and try to determine the combined effect of these structures and the larger scale magnetic field

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on the production of oscillatory motions in solar prominences.

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References

Anderson, L.S., Athay, R.G., 1989. Chromospheric and coronal heating. Astrophys. J. 336, 1089–1091.

Berger, M.A., Ricca, R.L., 1996. Topological ideas and fluid mechanics. Phys. Today 49 (12), 28–34.

Berger, T.E., Shine, R.A., Slater, G.L., et al., 2008. Hinode SOT observations of solar quiescent prominence dynamics. Astrophys. J. Lett. 676, L89–L92.

Chae, J., Ahn, K., Lim, E.-K., Choe, G.S., Sakurai, T., 2008. Persistent horizontal flows and magnetic support of vertical threads in a quiescent prominence. Astrophys. J. Lett. 689, L73–L76.

D'Alembert, 1747, Recherches sur la courbe que forme une corde tendu mise en vibration (Researches on the curve that a tense cord forms [when] set into vibration), Histoire de l'acadmie royale des sciences et belles lettres de Berlin, 3, pp. 214–219.

Engvold, O., 1976. The fine structure of prominence. Sol. Phys. 49, 283–295.

Engvold, O., Tandberg-Hanssen, E., Reichmann, E., 1985. Evidence for systematic flows in the transition region around prominences. Sol. Phys. 96, 35–51.

Kubota, J., Uesugi, A., 1986. The vertical motion of matter in a prominence observed on May 7, 1984, PASJ 38, pp. 903–909.

Kippenhahn, R., Schluter, A., 1957. Eine Theorie der solaren Filamente (A theory of solar filaments). Zeit. Fur. Astrophysik 43, 36–62.

Kuperus, M., Tandberg-Hanssen, T., 1967. The nature of quiescent solar prominences. Sol. Phys. 2 (1), 39–48.

Leroy, J.L., 1989. Observation of Prominence Magnetic Fields, Dynamics and Structures of Quiescent Prominences, ed. E.R. Priest, Kluwer Academic Publishers, Dordrecht, Holland, pp. 77–113.

Martres, M.J., Mein, P., Schmieder, B., Soru-Escaut, I., 1981. Structure and evolution of velocities in quiescent filaments. Sol. Phys. 69, 301–312.

Liggett, M., Zirin, H., 1984. Rotation in prominences. Sol. Phys. 91 (2), 259–267.

Lin, Y., Engvold, O., Wiik, J.E., 2003. Counter streaming in a large polar crown filament. Sol. Phys. 216 (1–2), 109–120.

Low, B.C., 1982. The vertical filamentary structures of quiescent prominences. Sol. Phys. 75, 119–131.

Mackay, D.H., Karpen, J.T., Ballester, J.L., Schmieder, B., Aulanier, G., 2010. Physics of solar prominences: II Magnetic structure and dynamics. Space Sci. Rev. 151 (4), 333–399.

Petrie, G.J.D., Low, B.C., 2005. The dynamical consequences of spontaneous current sheets in quiescent prominences. Astrophys. J. 159, 288–313.

Raadu, M.A., Kuperus, M., 1973. Thermal instability of coronal neutral sheets and the formation of quiescent prominences. Sol. Phys. 28, 77–94.

Saha, S., 2011. Differential Equations-A structured approach. Cognella Academic Press, San Diego, 2011, pp. 1–192.

Schmieder, B., Chandra, R., Berlicki, A., Mein, P., 2010. Velocity vectors of a quiescent prominence observed by Hinode/SOT and the MSDP (Meudon), A&A 514, A68.

Tsuneta, S., Ichimoto, K., Katsukawa, Y., Nagata, S., Otsubo, M., Shimizu, T., et al., 2008. The Solar Optical Telescope for the Hinode mission: an overview. Sol. Phys. 249 (2), 167–196.

Terradas, J., Soler, R., Luna, M., Oliver, R., Ballester, J.L., 2015. Morphology and dynamics of solar prominences from 3D MHD simulations. Astrophys. J. 799 (1), 94.

Terradas, J., Soler, R., Luna, M., Oliver, R., Ballester, J.L., Wright, A.N., 2016. Solar prominences embedded in flux ropes: morphological features and dynamics from 3D MHD simulations. Astrophys. J. 820 (2), 125.

Tandberg-Hanssen, E., 1995. The Nature of Solar Prominences. Dordrecht Kluwer.

Webb, D., Rust, D., Schmieder, B., 1998. New Perspectives on Solar Prominences. In: IAU Colloquium 167 – Proceedings of a Meeting held in Aussois, France, ASP Conference Series, San Francisco, USA, Vol. 150 (ISBN:1-886733-70-8).

Zirker, J.B., Engvold, O., Martin, S.F., 1998. Counter-streaming gas flows in solar prominences as evidence for vertical magnetic fields. Nature 396 (6710), 440–441.

Zirin, H., 1988. Astrophysics of the Sun. Cambridge Univ. Press, Cambridge, UK.